

§ 2.7: Combining Functions

Sums, Differences, Products, and Quotients

Two functions f and g can be combined to form new functions $f + g$, $f - g$, fg , and f/g . Here is how these new functions are created:

Algebra of Functions

Let f and g be functions with domains A and B . Then the functions $f + g$, $f - g$, fg , and f/g are defined as follows.

$$(f + g)(x) = f(x) + g(x) \quad \text{Domain } A \cap B$$

$$(f - g)(x) = f(x) - g(x) \quad \text{Domain } A \cap B$$

$$(fg)(x) = f(x)g(x) \quad \text{Domain } A \cap B$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad \text{Domain } \{x \in A \cap B : g(x) \neq 0\}$$

Example 1	Combinations of Functions and Their Domains
-----------	---

Let $f(x) = \frac{1}{x+3}$ and $g(x) = \sqrt{x}$.

(a) Find the functions $f + g$, $f - g$, fg , and f/g and their domains.

(b) Find $(f + g)(4)$, $(f - g)(4)$, $(fg)(4)$, and $\left(\frac{f}{g}\right)(4)$.

Composition of Functions

One more way to combine functions is through composition. Suppose $f(x) = \sqrt{x}$ and $g(x) = x^2 + 1$. We may define a function h as

$$h(x) = f(g(x)) = f(x^2 + 1) = \sqrt{x^2 + 1}.$$

Composition of Functions

Given two functions f and g , the **composition function** $f \circ g$ is defined by

$$(f \circ g)(x) = f(g(x)).$$

The domain of $f \circ g$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f .

Example 2	Finding the Composition of Functions
-----------	--------------------------------------

Let $f(x) = x^2$ and $g(x) = x - 3$.

- (a) Find the functions $f \circ g$ and $g \circ f$ and their domains.
- (b) Find $(f \circ g)(5)$ and $(g \circ f)(7)$.

Example 3	Finding the Composition of Functions
-----------	--------------------------------------

If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$, find the following functions and their domains.

(a) $f \circ g$

(b) $g \circ f$

(c) $f \circ f$

(d) $g \circ g$

Example 4	A Composition of Three Functions
-----------	----------------------------------

Find $f \circ g \circ h$ if $f(x) = \frac{x}{x+1}$, $g(x) = x^9$, and $h(x) = x + 1$.

Example 5	Recognizing a Composition of Functions
-----------	--

Given that $F(x) = \sqrt[3]{x^2 + 5}$, find two functions f and g so that

$$F(x) = (f \circ g)(x).$$

Example 6	Area of a Balloon
-----------	-------------------

A spherical weather balloon is being inflated. The radius of the balloon is increasing at the rate of 2 cm/s. Express the surface area of the balloon as a function of time t (in seconds).

Homework

Due: _____

$2 - 10$ (even), $18 - 22$ (even), $30 - 54$ (even), $58, 62$